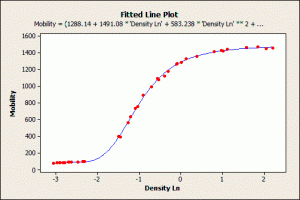
<https://statisticsbyjim.com/regression/r-squared-invalid-nonlinear-regression/>

<https://statisticsbyjim.com/regression/standard-error-regression-vs-r-squared/>

# R-squared Is Not Valid for Nonlinear Regression

By [Jim Frost](https://statisticsbyjim.com/author/statis11_wp/) [17 Comments](https://statisticsbyjim.com/regression/r-squared-invalid-nonlinear-regression/#comments)

Nonlinear [regression](https://statisticsbyjim.com/glossary/regression-analysis/) is an extremely flexible analysis that can fit most any curve that is present in your data. [R-squared](https://statisticsbyjim.com/glossary/r-squared/) seems like a very intuitive way to assess the goodness-of-fit for a regression model. Unfortunately, the two just don’t go together. R-squared is invalid for nonlinear regression.

Example of a nonlinear model that displays the relationship between density and electron mobility.

Some statistical software calculates R-squared for these models even though it is statistically incorrect. Consequently, it’s important that you understand why you should not trust R-squared for models that are not linear. In this post, I highlight research that shows you how assessing R-squared for nonlinear regression causes serious problems and leads you astray.

## Why Is R-squared Valid for Only Linear Regression?

In my post about [how to interpret R-squared](https://statisticsbyjim.com/regression/interpret-r-squared-regression/), I explain how R-squared is the following proportion:

{\displaystyle R^2 = \frac {\text{Variance explained by the model}}{\text{Total variance}}}

Furthermore, the variances always add up in a particular way:

Explained variance + Error variance = Total variance.

This arrangement produces an R-squared that is always between 0 – 100%.

That all makes sense, right? For linear models, this works out as you expect.

However, this math works out correctly only for linear regression models. In nonlinear regression, these underlying assumptions are incorrect. Explained variance + Error variance DO NOT add up to the total variance! The result is that R-squared isn’t necessarily between 0 and 100%. There are other problems with it as well.

This problem completely undermines R-squared in the context of nonlinear regression.

Keep in mind that I’m referring specifically to nonlinear models. R-squared is valid for linear models that use polynomials to model curvature.  If you’re not clear about the difference between these two types of models, read my post to learn [how to distinguish between linear and nonlinear regression](https://statisticsbyjim.com/regression/difference-between-linear-nonlinear-regression-models/).

## Specific Problems of Using R-squared with Nonlinear Regression

The general mathematic framework for R-squared doesn’t work out correctly if the regression model is not linear. Despite this issue, most statistical software still calculates R-squared for nonlinear models. This questionable practice can cause problems for you. Let’s see the ramifications!

Spiess and Neumeyer\* performed a simulation study to look at the [effect](https://statisticsbyjim.com/glossary/effect/) of using R-squared to assess the goodness-of-fit for models that are not linear. Their study ran thousands of simulations and found that R-squared leads you to draw false conclusions about which nonlinear models are best.

If you use R-squared for nonlinear models, their study indicates you will experience the following problems:

* R-squared is consistently high for both excellent and appalling models.
* R-squared will not rise for better models all of the time.
* If you use R-squared to pick the best model, it leads to the proper model only 28-43% of the time.

If you take all of these together, R-squared can’t differentiate between good and bad nonlinear models. It just doesn’t work. The authors go on to disparage the continuing practice of statistical software to calculate R-squared for nonlinear regression:

In the field of biochemical and pharmacological literature there is a reasonably high occurrence in the use of R2 as the basis of arguing against or in favor of a certain model. . . . Additionally, almost all of the commercially available statistical software packages calculate R2 values for nonlinear fits, which is bound to unintentionally corroborate its frequent use. . . . As a result from this work, we would like to advocate that R2 should not be reported or demanded in pharmacological and biochemical literature when discussing nonlinear data analysis.

If your statistical software calculates R-squared for nonlinear models, don’t trust it!

There are other goodness-of-fit measures you can use for regression models that are not linear. For instance, you can use( below) [the standard error of the regression](https://statisticsbyjim.com/regression/standard-error-regression-vs-r-squared/). For this statistic, smaller values represent better models.

If you’re learning regression, check out my [Regression Tutorial](https://statisticsbyjim.com/regression/regression-tutorial-analysis-examples/)!

### Reference

Spiess, Andrej-Nikolai, Natalie Neumeyer. An evaluation of R2 as an inadequate measure for nonlinear models in pharmacological and biochemical research: a Monte Carlo approach. BMC Pharmacology. 2010; 10: 6.

Dear Mr Frost,  
Thank you for your article, it is really enlightening. I just have a question regarding the reference you used. According to the paper, the residual variance (very similar to MSE) is just as bad as the R2. That said, is it really OK to use RMSE to measure a model performance? The non-linear model I am using is called Gradient Boosting Machine (clearly highly non linear).  
Thank you very much,  
Rodrigo

[December 11, 2020 at 9:03 pm](https://statisticsbyjim.com/regression/r-squared-invalid-nonlinear-regression/#comment-8056)

Hi Rodrigo,

I always recommend using the [standard error of the regression (S)](https://statisticsbyjim.com/regression/standard-error-regression-vs-r-squared/), which is the square root of the adjusted mean square error. The adjustment is the same as adjusted R-squared, which is based on the number of predictors. Using either adjusted R-squared or the standard error of the regression, it’s possible to add predictors and obtain a worse fit. In that way, S is better than R-squared. And, it’s valid to use in nonlinear regression.

Below is the equation for the adjusted mean squared error. Take the square root of it to obtain the standard error of the regression (S).  
Equation for adjusted mean square error.  
Where n is the number of observations and p is the number of coefficients in the model not counting the constant.

SSE also called RSS (residual SS), MSE also called RMS (Residual MS)

Hi Jim,

I’m a complete novice and need to fit some data for a project, but am quite stuck. I’ve read through your explanations of non-linear curve fitting (really really helpful!!!), and I’ve managed to fit a curve using an exponential model that looks quite good.

However, I am using the program OriginPro, and can’t seem to find a value for the standard error of the regression (S). I don’t know if this is just me being stupid, but I’m sure I can’t find a value. I have the following information though, would it be possible to work it out from this? (I won’t list all the numbers, I’ll just put ‘value’ in place).

Plot of X vs Y, with error in Y  
Model: ExpAssoc  
Equation: y = y0 + A1\*(1-exp(-x/t1)) + A2\*(1-exp(-x/t2))  
Plot: B  
For y0,A1,A2,t1 and t2, all have a value +/- error in value  
For Plot (B), I have the following info:  
Number of data points = 7  
Degrees of freedom = 2  
Reduced Chi squared: 0.1021  
Residual sum of squares: 0.2042  
R squared (COD): 0.99976  
Adjusted R squared: 0.99928  
Fit status: succeeded (100)

If anyone could let me know if I’ve done something wrong in the fitting and that is why I can’t find an S value, or if I’m missing something entirely, that would be really really helpful!

[February 23, 2021 at 3:18 pm](https://statisticsbyjim.com/regression/standard-error-regression-vs-r-squared/#comment-8539)

Hi Natasha,

I have never use OriginPro. However, from your output, I can see that it does not include the standard error of the regression (S). Fortunately, we can calculate it using the statistics you provide. By the way, OriginPro should not provide [R-squared for nonlinear regression because it is invalid in that context](https://statisticsbyjim.com/regression/r-squared-invalid-nonlinear-regression/). It’s great that you’re looking into S!

S is the square root of the MSE. And MSE = SS Error/DF error.

So, we’ll take 0.2042 (your residual sum of squares, aka SSE) and divide by 4 (DF error), which equals 0.05105 (MSE). In case you’re wondering how I obtained the 4 DF Error. The total DF = N – 1 = 6. Error DF = Total DF – Regression DF, or 6 – 2 = 4.

The MSE = 0.05105 and we just need to take the square root of that to get S! So, S = 0.2259

I hope that helps!

Also see:

<https://blog.minitab.com/en/adventures-in-statistics-2/why-is-there-no-r-squared-for-nonlinear-regression>